# LECTURE-05

Let us consider the DE

$$M(x,y)\,dx + N(x,y)\,dy = 0 \quad \text{——— (1)}$$

If the DE (1) is <u>exact</u> i.e $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Then its solution is given by,

$$\int M(x,y)\,dx + \int N(x,y)\,dy = C \quad \text{——— (2)}$$

(Y-CONST.) (Term not cont. x in N)

If the DE (1) is <u>not exact</u> i.e $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ then we have to find out a function $\mu(x,y)$ such that after multiplying it into both sides of equation (1) it becomes exact. Then we can use equation (2) to get the solution.

This function $\mu(x,y)$ is called an integrating factor of equation (1).

Hence the role of an integrating factor is to convert a non exact DE into an exact DE after multiplying it into both sides of the given DE.

NOTE:-

One DE can have infinite number of integrating factor.

## Methods for finding out integrating factor

**RULE-1**

If $\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = f(x)$ then the integrating factor is $e^{\int f(x)\,dx}$.

**RULE-2**

If $\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M} = f(y)$ then the integrating factor is $e^{-\int f(y)\,dy}$

**NO-25-TO-30, PAGE-32, E. KREYSZIG (8TH EDITION)**

~~25. $\sin y\,dx + \cos y\,dy = 0$~~

Show that the given function is an integrating factor and solve

25. $\sin y\,dx + \cos y\,dy = 0$ —— (1) $e^x$.

Multiplying $e^x$ into both sides of equation (1)

$e^x \sin y\,dx + e^x \cos y\,dy = 0$ —— (2)

Now, $M(x,y) = e^x \sin y$ $\quad N(x,y) = e^x \cos y$

$\frac{\partial M}{\partial y} = e^x \cos y$ $\quad \frac{\partial N}{\partial x} = e^x \cos y$

Thus, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

So, equation (1) is an exact DE after multiplying it by $e^x$.

Thus $e^x$ is an integrating factor of equation (1)

The solution is given by,

$$\int M(x,y)\,dx + \int N(x,y)\,dy = C$$

(Y-const.) (Term not cont. x in N)

$$\Rightarrow \int e^x \sin y\,dx = C$$

(Y-const.)

$$\Rightarrow e^x \sin y = C.$$

29. $(2y+xy)\,dx + 2x\,dy = 0$ —— (1) $\quad \frac{1}{xy}$

Multiplying $\frac{1}{xy}$ into both sides of equation (1)

$$\left(\frac{2}{x}+1\right)dx + \frac{2}{y}\,dy = 0 \quad \text{—— (2)}$$

now, $M(x,y) = \frac{2}{x}+1$ & $N(x,y) = \frac{2}{y}$

$$\frac{\partial M}{\partial y} = 0 \qquad \frac{\partial N}{\partial x} = 0$$

Thus, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

So, equation (1) is an exact DE after multiplying it by $\frac{1}{xy}$

Thus, $\frac{1}{xy}$ is an integrating factor of equation (1)

The solution is given by,

$$\int M(x,y)\,dx + \int N(x,y)\,dy = C$$

(Y-const.) (Term not cont. x in N)

$$\Rightarrow \int \left(\frac{2}{x}+1\right)dx + \int \frac{2}{y}\,dy = C$$

(Y-const.)

$\Rightarrow 2\ln|x| + x + 2\ln|y| = C$

$\Rightarrow \ln|x^2| + x + \ln|y^2| = C$

$\Rightarrow \ln|x^2y^2| + x = C$

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Find an integrating factor and solve.

32. $2xy\,dx + 3x^2\,dy = 0$ ——— (1)

Here, $M(x,y) = 2xy$ $\quad N(x,y) = 3x^2$

$\frac{\partial M}{\partial y} = 2x$ $\quad \frac{\partial N}{\partial x} = 6x$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M} = \frac{2x - 6x}{2xy} = \frac{-4x}{2xy} = -\frac{2}{y} = f(y)$$

Thus, the integrating factor is

$$e^{\int f(y)\,dy} = e^{-\int -\frac{2}{y}\,dy} = e^{2\ln|y|} = e^{\ln|y^2|} = y^2.$$

Multiplying the I.F into both sides of equation (1)

$2xy^3\,dx + 3x^2y^2\,dy = 0$

which is an exact DE

The solution is given by

$$\int M(x,y)\,dx + \int N(x,y)\,dy = C$$

(y-CONST.) (Term not cont. x in N)

$\Rightarrow \int_{(y\text{-CONST.})} 2xy^3\,dx = C$

$\Rightarrow x^2y^3 = C$

$$\frac{-4x}{2xy} = -\frac{2}{y} = f(y)$$

$$\frac{-4x}{3x^2} = -\frac{4}{3x} = f(x)$$

34. $2\cos y\,dx = \sin y\,dy$

$\Rightarrow 2\cos y\,dx - \sin y\,dy = 0$ ——— (1)

Here, $M(x,y) = 2\cos y$ $\quad N(x,y) = -\sin y$

$\frac{\partial M}{\partial y} = -2\sin y$ $\quad \frac{\partial N}{\partial x} = 0$

$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{-2\sin y}{2\cos y} = -\tan y = f(y)$

$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{-2\sin y - 0}{-\sin y} = \frac{-2\sin y}{-\sin y} = 2 = f(x)$

Thus, the integrating factor is

$e^{\int f(x)\,dx} = e^{\int 2\,dx} = e^{2x}$

Multiplying the integrating factor into both sides of equation (1)

$2e^{2x}\cos y\,dx - e^{2x}\sin y\,dy = 0$

which is an exact DE.

The solution is given by

$\int M(x,y)\,dx + \int N(x,y)\,dy = C$

(y-CONST) (Term not cont. x in N)

$\Rightarrow \int 2e^{2x}\cos y\,dx = C$

(y-const)

$\Rightarrow e^{2x}\cos y = C.$

35. $2x \tan y \, dx + \sec^2 y \, dy = 0$ ——— (1)

Here, $M(x,y) = 2x \tan y$ $\quad N(x,y) = \sec^2 y$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{2x\sec^2 y - 0}{\sec^2 y} = 2x = f(x)$$

Thus, the integrating factor is

$$e^{\int f(x)\,dx} = e^{\int 2x\,dx} = e^{x^2}$$

Multiplying the I.F into both sides of equation (1)

$$2x\, e^{x^2} \tan y \, dx + e^{x^2} \sec^2 y \, dy = 0$$

which is an exact DE.

$$\frac{2x\sec^2 y}{\sec^2 y} = 2x = f(x)$$

$$\frac{2x\sec^2 y}{2x\tan y} = \frac{\sec^2 y}{\tan y} = f(y)$$

The solution is given by,

$$\int M(x,y)\,dx + \int N(x,y)\,dy = c$$

(Y-CONST.) (Term not cont. $x$ in $N$)

$$\Rightarrow \int 2x\, e^{x^2} \tan y \, dx = c$$

(Y-CONST)

$$\Rightarrow \tan y \int 2x\, e^{x^2} dx = c$$

Put $x^2 = p$

$2x\,dx = dp$

$$\Rightarrow \tan y \int e^p \, dp = c$$

$$\Rightarrow e^p \tan y = c$$

$$\Rightarrow e^{x^2} \tan y = c.$$